$\square$ Code No. : 21912
VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD

# M.Tech. (CSE: CBCS) I-Semester Main Examinations, December-2018 <br> Mathematical Foundations of Computer Science 

Time: 3 hours
Max. Marks: 60
Note: i) Answer ALL questions in Part-A and any FIVE from Part-B
ii) Use of Normal, $t, F, x^{2}$ - distribution tables are permitted.
Q.No.
Stem of the question
Part- (10 $\times 2=20 \mathrm{Marks})$
Give example.

1. What is random variable? Give example.
2. How mean and variance are found using moment generating function of a distribution?
3. State Central Limit theorem.
4. Find variance of Uniform Distribution.
5. Define Random Sampling and give example.
6. What do you mean by an Estimator?
7. Distinguish between Parameters and Statistics.
8. State Level of Significance.
9. Explain Regression Analysis.
10. Explain method of Least Squares.

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\text { Part-B }(5 \times 8=40 \mathrm{Marks})
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11. a) A continuous RV X has a pdf $f(x)=k x^{2} e^{-x} ; x \geq 0$. Find k , mean and variance.
b) For the joint probability distribution of two random variables X and Y given below:

| $\mathbf{X}$ | $\mathbf{1}$ | 2 | 3 | 4 | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $4 / 36$ | $3 / 36$ | $2 / 36$ | $1 / 36$ | $10 / 36$ |
| 2 | $1 / 36$ | $3 / 36$ | $3 / 36$ | $2 / 36$ | $9 / 36$ |
| 3 | $5 / 36$ | $1 / 36$ | $1 / 36$ | $1 / 36$ | $8 / 36$ |
| 4 | $1 / 36$ | $2 / 36$ | $1 / 36$ | $5 / 36$ | $9 / 36$ |
| Total | $11 / 36$ | $9 / 36$ | $7 / 36$ | $9 / 36$ | 1 |

Find: i) The marginal distributions of X and Y , and
ii) Conditional distribution of X given the value of $\mathrm{Y}=1$ and that of Y given the value of $\mathrm{X}=2$.
12. a) In a distribution exactly normal, $10.03 \%$ of the items are under 25 kilogram weight and $89.97 \%$ of the items are under 70 kilogram weight. What are the mean and standard deviation of the distribution?
b) The daily consumption of milk in a city, in excess of 20,000 litres, is approximately distributed as a Gamma variate with parameters $\mathrm{a}=\frac{1}{1000}$ and $\lambda=2$. The city has a daily stock of 30,000 litres. What is the probability that the stock is insufficient on a particular day?
13. a) Let $\mathrm{X}_{1}, \ldots \mathrm{X}_{\mathrm{n}}$ be a random sample of size n from a normal distribution with known variance. Obtain the maximum likelihood estimator of $\mu$.
b) Let $x_{1}, \ldots x_{n}$ be the observed values of a random sample of size $n$ from the exponential distribution $f(x ; \beta)=\beta^{-1} e^{-x / \beta}$ for $\mathrm{x}>0$. Find the maximum likelihood estimator of $\beta$.

## 14. a) A random sample of 10 boys had the following:

I.Q.'s: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100.

Do these data support the assumption of a population mean I.Q. of 100 ? Find a reasonable range in which most of the mean I.Q. values of samples of 10 boys lie.
b) Two sample polls of votes for two candidates A and B for public office are taken, one from among the residence of rural areas. The results are given in the table. Examine whether the nature of the area is related to voting preference in this election.

| Area | Votes for |  | Total |
| :--- | :---: | :---: | :---: |
|  | A | B | 1000 |
| Rural | 620 | 380 | 1000 |
| Urban | 550 | 450 | 1000 |
| Total | 1170 | 830 | 2000 |

15. a) The following are data on the number of twists required to break a certain kind of forged alloy bar and the percentages of two alloying elements present in the metal:

| Number of Twists <br> $y$ | Percentage of element A <br> $x_{1}$ | Percentage of element B <br> $x_{2}$ |
| :---: | :---: | :---: |
| 41 | 1 | 5 |
| 49 | 2 | 5 |
| 69 | 3 | 5 |
| 65 | 4 | 5 |
| 40 | 1 | 10 |
| 50 | 2 | 10 |
| 58 | 3 | 10 |
| 57 | 4 | 10 |
| 31 | 1 | 15 |
| 36 | 2 | 15 |
| 44 | 3 | 15 |
| 57 | 4 | 15 |
| 19 | 2 | 20 |
| 31 | 3 | 20 |
| 33 | 4 | 20 |
| 43 | 2 | 20 |

Fit a least squares regression plane and use its equation to estimate the number of twists required to break one of the bars when $x_{1}=2.5$ and $x_{2}=12$.
b) The following are the numbers of minutes it took 10 machines to assemble a piece of machinery in the morning, $x$, and in the late afternoon, $y$ :

| $\mathbf{x}$ | y |
| :---: | :---: |
| 11.1 | 10.9 |
| 10.3 | 14.2 |
| 12.0 | 13.8 |
| 15.1 | 21.5 |
| 13.7 | 13.2 |
| 18.5 | 21.1 |
| 17.3 | 16.4 |
| 14.2 | 19.3 |
| 14.8 | 17.4 |
| 15.3 | 19.0 |

Calculate coefficient of correlation.
$\begin{array}{lll}4 & 3 & 4\end{array}$
$4 \quad 4 \quad 4 \quad 3$
$4 \quad 4 \quad 5 \quad 3$

433
16. a) For the bivariate probability distribution of X and Y ,

| $\mathbf{X}$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | $1 / 32$ | $2 / 32$ | $2 / 32$ | $3 / 32$ |
| 1 | $1 / 16$ | $1 / 16$ | $1 / 8$ | $1 / 8$ | $1 / 8$ | $1 / 8$ |
| 2 | $1 / 32$ | $1 / 32$ | $1 / 64$ | $1 / 64$ | 0 | $2 / 64$ |

find: i) $\mathrm{P}(\mathrm{X} \leq 1, \mathrm{Y}=2)$,
ii) $\mathrm{P}(\mathrm{X} \leq 1)$,
iii) $\mathrm{P}(\mathrm{Y}=3)$, and
iv) $\mathrm{P}(\mathrm{X}<3, \mathrm{Y} \leq 4)$.
b) X is a normal variate with mean 30 and S.D. 5. Find the probabilities that
i) $26 \leq X \leq 40$,
ii) $X \geq 45$, and
iii) $|X-30|>5$.
17. Answer any two of the following:
a) Explain how to generate Estimators by using maximum Likelihood method.
b) The following random samples are measurements of the heat producing capacity (in millions of calories per ton) of specimens of coal from two mines:

| Mine 1: | 8,260 | 8,130 | 8,350 | 8,070 | 8,340 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mine 2: | 7,950 | 7,890 | 7,900 | 8,140 | 7,920 | 7,840 |

Use the 0.01 level of significance to test whether the difference between the Means of these two samples is significant.
c) The expected remaining life of an electronic part is believed to be related to the age of the part. The ages of 10 of these parts that were in use on a certain date were recorded in operating hours. When each part burned out, the elapsed time was recorded. The results were as follows:

| Age of Part (in hrs): | 40 | 65 | 90 | 5 | 30 | 10 | 80 | 85 | 70 | 25 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Remaining Life (in hrs): | 30 | 20 | 10 | 80 | 40 | 65 | 15 | 15 | 20 | 50 |

Determine the coefficients of regression and regression curve using the nonlinear form $Y=a b^{X}$.

M: Marks; L: Bloom's Taxonomy Level; CO: Course Outcome; PO: Programme Outcome

| S. No. | Criteria for questions | Percentage |
| :---: | :--- | :---: |
| 1 | Fundamental knowledge (Level-1 \& 2) | 52.5 |
| 2 | Knowledge on application and analysis (Level-3 \& 4) | 42.5 |
| 3 | *Critical thinking and ability to design (Level-5 \& 6) <br> (*wherever applicable) | 5.0 |

